4. Stresses in Flexible pavement

Assistant Professor Dr. Weerakaset Suanpaga (D.Eng)
Department of Civil Engineering
KASETSART UNIVERSITY

Chapter 4. Introduction to stresses in flexible pavement

- Vehicular wheel loads => induce stresses in pavement structure.
- The stresses => create strains => accumulate resulting excessive plastic strain (cracking, rutting, and roughness)
- Pavement damages which reduce pavement riding quality and are the major causes of pavement functional failure.
- The knowledge of stresses and strains analysis is thus important for the design of pavement structures.

4.1 Introductions to concept of stress and strain in continuum mechanics

- Continuum mechanics is the theory for the analysis of stresses and strain in a deformable body (solid and liquid).
- It is applied only to continuous media (continuum) in the macroscopic meaning. (Materials are discontinuous in molecular scale).
- The principal assumption of Continuum mechanics requires that for a given corps under course of deformation, two points initially neighboring would remain neighboring after deformations.

4.2 Boussinesq’s Theory

- Ideal Masses => analysis soil reaction under load by using Mathematical Theory of Elasticity
- Assumption
  1. Soil is in elasticity material, Homogeneous, Isotropic, Semi-infinite Medium
  2. Soil prosperities following by Hook’s law

4.2 Boussinesq’s Theory-con’t

- 3. Unit weight of soil is zero ($\gamma = 0$), consider only load action over the soil surface
- 4. No stress born before load acting
- 5. Poisson’s Ratio ($\mu$) is constant due to load transfer; normally using $\mu = 0.5$
- 6. Linear Stress function distribution
- 7. Vertical stress is Symmetry
Vertical stress at point $z$ due to Point Load

\[
\sigma_z = \frac{3P}{2\pi z^2 [1 + (r/z)^2]^{5/2}}
\]

\[
\sigma_z = \frac{K P}{z^2}
\]

\[
K = \frac{3}{2\pi [1 + (r/z)^2]^{5/2}}
\]

Vertical stress at point $z$ due to contact area

Equal radii $a$

\[
d\sigma_z = \frac{3p}{2\pi z^2} \left[ \frac{1}{1 + (r/z)^2} \right]^{5/2} dA
\]

\[
\sigma_z = p \left[ 1 - \frac{z^3}{(a^2 + z^2)^{3/2}} \right]
\]

Horizontal stress

\[
\sigma_x = \frac{E \mu}{2} \left[ 1 + 2\mu - \frac{2(1+\mu)z}{(a^2 + z^2)^2} + \frac{z^3}{(a^2 + z^2)^{3/2}} \right]
\]

Relation between $\sigma_z$ and $z$

\[
\sigma_z = p \left[ 1 - \frac{z^3}{(a^2 + z^2)^{3/2}} \right]
\]

Deflection

Deflection due to wheel load by applying flexible plate, the maximum deflection at the center of loads

\[
deflection \Delta = \frac{3pa^2}{2E(a + z^2)^3}
\]
Deflection behavior due to rigid plate load

- Minimum deflection at edge of rigid plate
- Maximum deflection at center of rigid plate

Stress in horizontal distance \( r \)

\[
\sigma_r = \frac{p \cdot a}{(2a^2 - r^2)^{1.5}}
\]

Deflection

\[
\Delta = \frac{\pi(1 - \mu^2)pa}{2E}
\]

Elastic Deformation under circular area load

Considering the small volume of soil under circular area load at any depth \( z \) from the surface then Elastic Strain

Elastic Strain \( \delta = \frac{1}{E} \left[ \sigma_r - 2 \mu \sigma_a \right] \)

\[
\Delta = \frac{p}{E} \left[ \frac{(2-2\mu^2)(a^2 + z^2)^{1.5} - (1 + \mu^2)z^2}{(a^2 + z^2)^{1.5}} + (\mu + 2\mu^2 - 1)z \right]
\]

Given \( \mu = 0.5 \)

\[
\Delta = \frac{3pa^2}{2E(a^2 + z^2)^{1.5}}
\]

Deflection under circular area load

\[
\Delta = \pi(1 - \mu^2)pa/2E
\]

\[
\Delta = 1.18pa/E, \text{ } (\mu = 0.5)
\]

When \( r \) = distance from centre of circular area load

\( a \) = radius in radii unit

\( p \) = average stress

Example

4.3 Burmister’s Theory

Two layer systems were presented by Burmister, the solutions of stresses and deflections under the center of circular load of the two-layer system by using assumption

1. Soil is homogenous, Isotropic and Elastic
2. Definite in depth and Infinite in the lateral direction
3. This theory can be used Boussinesq’s Theory apply in each layers.
4. NO shear stress between each contact layers

Deflection (Two-layer systems)

- Flexible Plate \( \Delta = 1.5paf_z/E_2 \) (flexible pavement)
- Rigid Plate \( \Delta = 1.18paf_z/E_2 \) (concrete pavement)

Given

\( p \) = stress pressure on circular area

\( a \) = radius of circular load

\( E_2 \) = modulus of elasticity of last layer of soil

\( F_2 \) = factor depended on \( E_2/E_1 \) and \( z/a \) see figure 2.14
Deflection (Two-layer systems)

- $F_2$ is the deflection factor, a function of layer modulus ratio, $(E_2/E_1)$ and the layer depth in multiple of contact radius is shown in figure 2.14.
- In this figure, the values of $E_2/E_1$ are shown on the curve.
- And $E_1$ represents the modulus of the upper layer.
- Where $E_2$ as the modulus of half space.

Method to calculate $F_2$

- $F_2$ depended on $E_2, E_1$ that can be determine from Plate Baring 2 times (first test on subgrade), the result can be determine $\Delta_1$.
- And can be calculate $E_2$ from above.
- Second Plate Baring test on pavement structure, the result can be determine $F_2$ and $E_1$.
4.4 Three-Layer System

- The solution for vertical stress was given by Peattie. The horizontal stress solution was obtained from John.
- The problem treated is the axi-symmetric type so the stress tensors reduce to only 4-components: the vertical normal stress, the horizontal radial normal stress, the circumferential normal and the shearing stress.

\[ \sigma_v \] The vertical normal stress,
\[ \sigma_r \] The horizontal radial normal stress,
\[ \sigma_\theta \] The circumferential normal
And \[ \tau \] the shearing stress.

The solution is given at the point on the axis of symmetry where \( \sigma_\theta \) vanishes and \( \sigma_r \) equal to \( \sigma_v \). The stresses could be determined using tables and charts and with given symbols \( a_i \).

\[ \sigma_v = \text{vertical stress at interface 1} \]
\[ \sigma_r = \text{radial stress at interface 2} \]
\[ \sigma_\theta = \text{radial stress at bottom of layer 1} \]
\[ \sigma_v = \text{radial stress at top of layer 3} \]

The following parameters are defined as:

\[ k_1 = \frac{a_1}{a_2} \]
\[ k_2 = \frac{a_2}{a_3} \]
\[ a_1 = \frac{a_1}{a_2} \]
\[ H = \frac{a_3}{a_2} \]

Where \( a_i, (a_2-R_i) \) are read from charts and table, (Figure 2.18 and Table 2.3). The corresponding strains can be computed from constitutive law equation.

It is important to be noted here that at the interfaces the quantities which are continuous across are the normal stress, the shearing stress and the displacements \( (u,v,w) \), but not the radial stress. (The horizontal displacements are equal, the radial stresses are determined by the relevant elastic modulus of each layer)

\[ \varepsilon_r = \frac{1}{E} (\sigma_r - v(\sigma_v + \sigma_\theta)) \]
\[ \varepsilon_\theta = \frac{1}{E} (\sigma_\theta - v(\sigma_v + \sigma_\theta)) \]
\[ \varepsilon_v = \frac{1}{E} (\sigma_v - v(\sigma_v + \sigma_\theta)) \]
At the point on the axis of symmetry, \( (\sigma_{yy} = \sigma_{xx}) \) and for \( v = 0.3 \), the critical strains (\( \varepsilon_\text{m}, \varepsilon_\text{m} \)) are simplified to:

\[
\begin{align*}
\varepsilon_\text{m} &= \frac{1}{2E} (\sigma_{yy} - \sigma_{xx}) \\
\varepsilon_\text{m} &= \frac{1}{2E} (\sigma_{yy} - \sigma_{xx})
\end{align*}
\]

Example of Three – Layer Systems

Three-layer pavement system showing location of stress solutions presented in text.

\[ \text{Layer 2.10} \quad \text{and up to 3 mm} \]

<table>
<thead>
<tr>
<th>( z_i )</th>
<th>( \sigma_{i1} )</th>
<th>( \sigma_{i1} / \sigma_{i1} )</th>
<th>( \epsilon_{i1} )</th>
<th>( \epsilon_{i1} )</th>
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<tr>
<td>0.1</td>
<td>0.63215</td>
<td>3.55</td>
<td>2.63</td>
<td>-6.92</td>
</tr>
<tr>
<td>0.2</td>
<td>1.83706</td>
<td>10.32</td>
<td>2.63</td>
<td>-7.69</td>
</tr>
<tr>
<td>0.4</td>
<td>3.68770</td>
<td>21.73</td>
<td>2.63</td>
<td>-19.10</td>
</tr>
<tr>
<td>0.8</td>
<td>5.50796</td>
<td>30.95</td>
<td>2.63</td>
<td>-28.32</td>
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<tr>
<td>1.6</td>
<td>4.28281</td>
<td>23.86</td>
<td>2.63</td>
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</tr>
<tr>
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<td>1.95404</td>
<td>11.40</td>
<td>2.63</td>
<td>-8.47</td>
</tr>
</tbody>
</table>

Note: The results are rounded to three significant figures.
Reference:
จิรพัฒน์ โชติกไกร, การออกแบบทาง, ภาควิชาวิศวกรรมโยธา คณะวิศวกรรมศาสตร์ มหาวิทยาลัยเกษตรศาสตร์, 2550

Table 4.3 three-layer stress factors (After Jones)

| Layer | Stress Factors | Reference | Question? | Thank you for your kind attention |